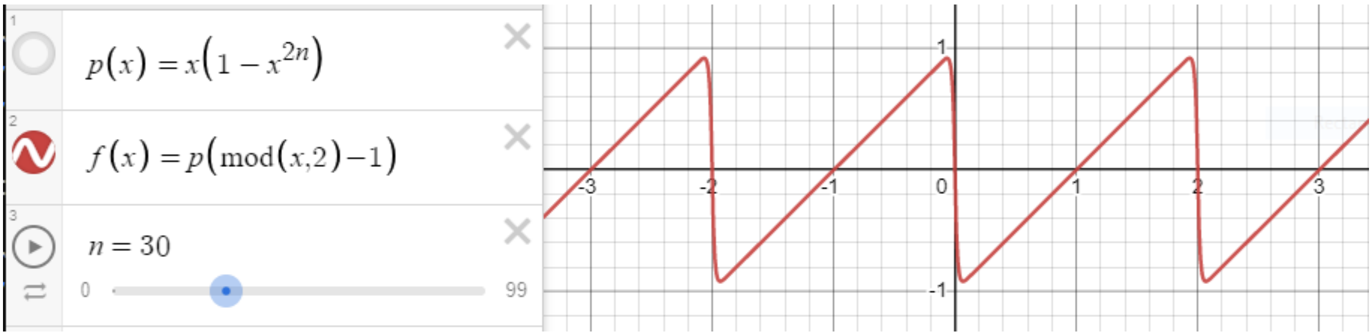
The following polynomial gives a single 'tooth' on domain $[-1,1]$. The higher n, the sharper the peak.

$$p(x) = x\left(1 - x^{2n}\right)$$

Repeat the function to extend the domain to $\mathbb{R}$:

$$f(x) = p((x \bmod 2) - 1)$$

A screenshot from [Desmos](https://www.desmos.com):



Note: f and f' are continuous, but the second derivative is not continuous.

As for efficiency: n is integer, so x^{2n} can be calculated with a sequence of $\mathcal{O}(\log n)$ multiplications.

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edited Aug 9 at 19:23

answered Aug 9 at 18:37

Ruud Helderman

136 5

In case anyone needs a smooth sawtooth function that is *infinitely differentiable* (C^∞): there are various ways to construct such a function, typically by combining trigonometric functions.

One way is to start with a smooth square wave like:

$$s(x) = \tanh(n \cos x)$$

or:

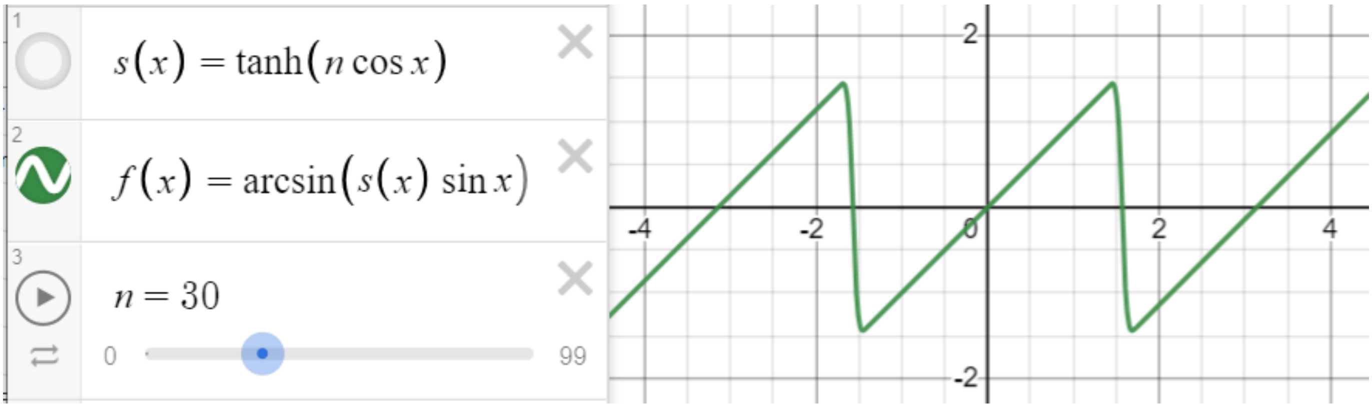
$$s(x) = \frac{2}{\pi} \arctan(n \cos x)$$

(the higher n, the sharper the edges)

Let the square wave flip a triangle wave to produce a sawtooth:

$$f(x) = \arcsin(s(x) \sin x)$$

Screenshot from Desmos:



Obviously, this is likely to be more computationally expensive than the polynomial in my other answer. But that one, being a piecewise function, offers only C^1 continuity.

Other suggestions can be found here: https://mathematica.stackexchange.com/q/38293

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